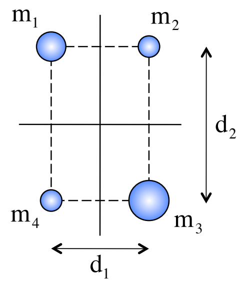
Problem 10.25

The particles are connected to each other via massless, rigid bars. The system rotates about the z-axis (coming out of the page) with angular speed of 6.00 rad/sec.



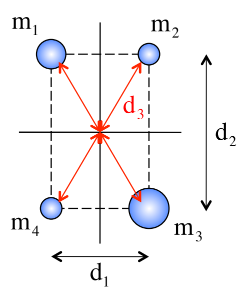
a.) Derive an expression for the system's *moment of inertia* about the axis of rotation.

When dealing with discrete masses, the *moment of inertia* is defined as:

$$I_z = \sum m_i r_i^2$$

This should look familiar. With the exception of the r^2 term, it is very similar to the *center of mass* expression. And just as was the case with that expression, it is asking you to do something that is very simple: move out away from the axis in question until you find some mass. Multiply that mass by the distance between it and the axis *quantity squared*. Do that for all masses, then sum.

Noting that the distance between ALL the masses and the z-axis is:

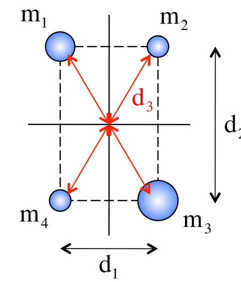


$$\begin{aligned} d_3 &= \left(\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2\right)^{1/2} \\ &= \frac{1}{2}\left(d_1{}^2 + d_2{}^2\right)^{1/2} \\ &= \frac{1}{2}\left((4.00\text{ m})^2 + (6.00\text{ m})^2\right)^{1/2} \\ &= 3.61\text{ m} \end{aligned}$$

So:

$$\begin{aligned} I_z &= \sum m_i r_i^2 \\ &= m_1 d_3{}^2 + m_2 d_3{}^2 + m_3 d_3{}^2 + m_3 d_3{}^2 \\ &= d_3{}^2 (m_1 + m_2 + m_3 + m_4) \\ &= (3.61\text{ m})^2 \left((3.00\text{ kg}) + (2.00\text{ kg}) + (4.00\text{ kg}) + (2.00\text{ kg})\right) \\ &= 143\text{ kg} \bullet \text{m}^2 \end{aligned}$$

b.) The rotational kinetic energy?



$$\begin{aligned} KE_{rot} &= \frac{1}{2} I_z \omega^2 \\ &= \frac{1}{2}\left(143\text{ kg} \bullet \text{m}^2\right)\left(6.00\text{ rad/s}\right)^2 \\ &= 2.57\text{x}10^3\text{ J} \end{aligned}$$